Uncovering the multiscale dynamics of temporal networks

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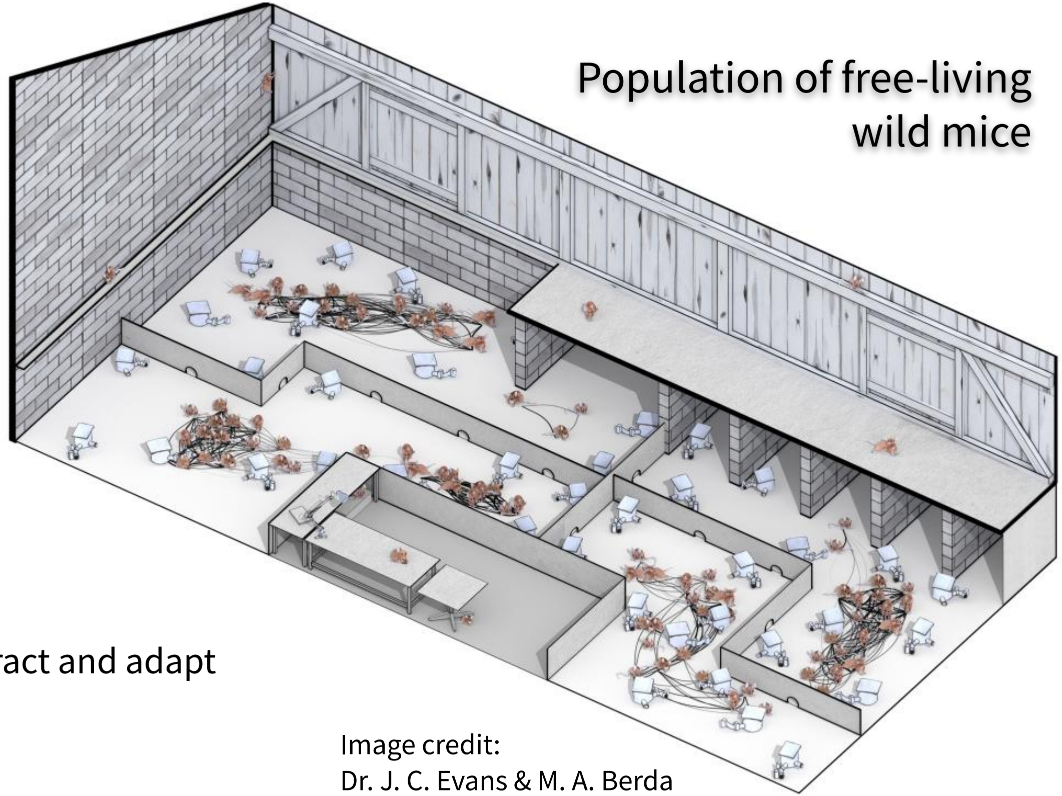
complenet

How to capture the temporal complexity of real systems?

How do social groups interact and adapt to changes?



Population of free-living wild mice

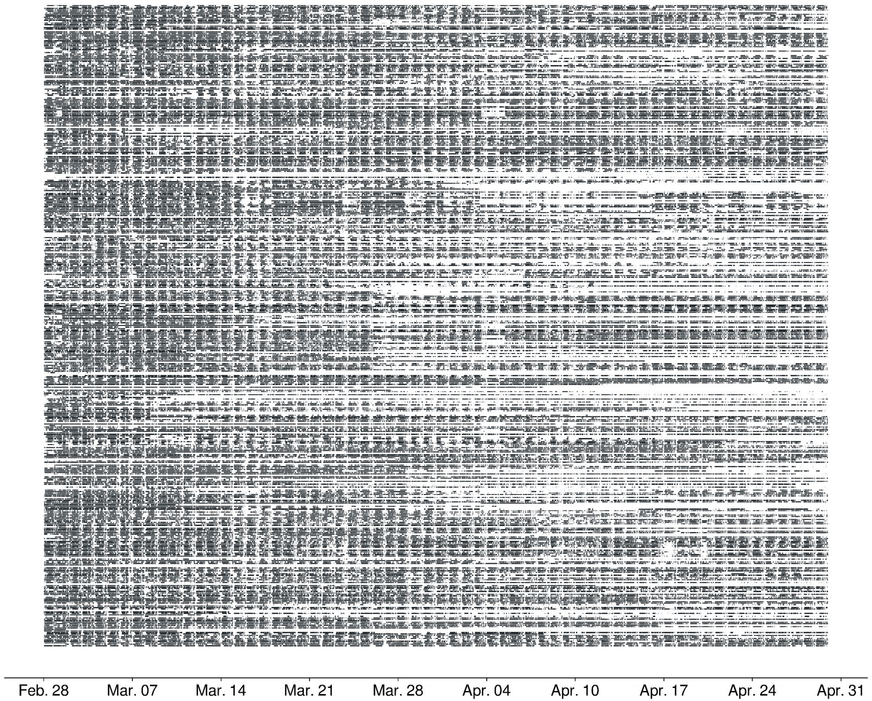
Image credit:
Dr. J. C. Evans & M. A. Berda

Prof. Barbara König

Prof. Anna Lindholm

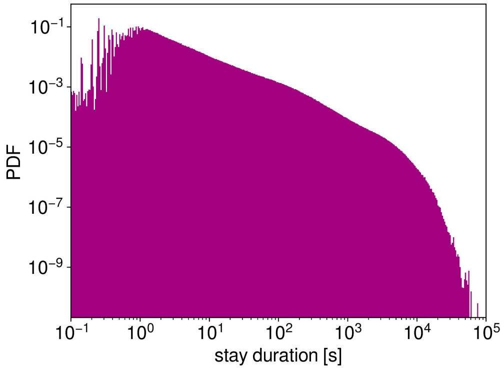
A complex dynamics due to multiple temporal processes

Mice activity in the barn



Simultaneous processes with different temporal characteristics

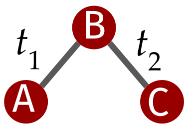
- Circadian activity
- Adaptation (seasonal)
- Competition
- Cooperation
- …



Temporal networks to capture time-resolved interactions

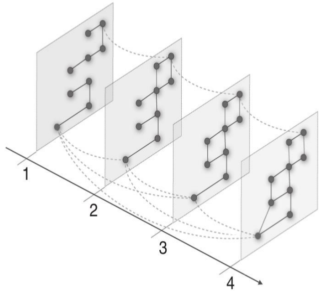
Allows to represent complex temporal patterns not captured by static networks:

- Burstiness
- Memory
- Non-stationarity



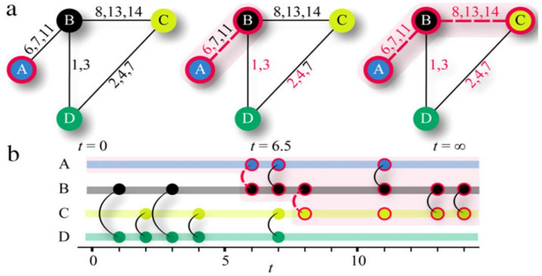
Asymmetric temporal paths

Snapshot sequence:
- discrete time
- sequence of graphs

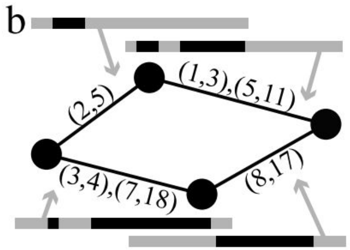


P. J. Mucha *et al.*, Science. 328, 876 (2010).

Contact sequence:
- continuous time
- instantaneous edges



Interval graphs:
- continuous time
- edges have a duration

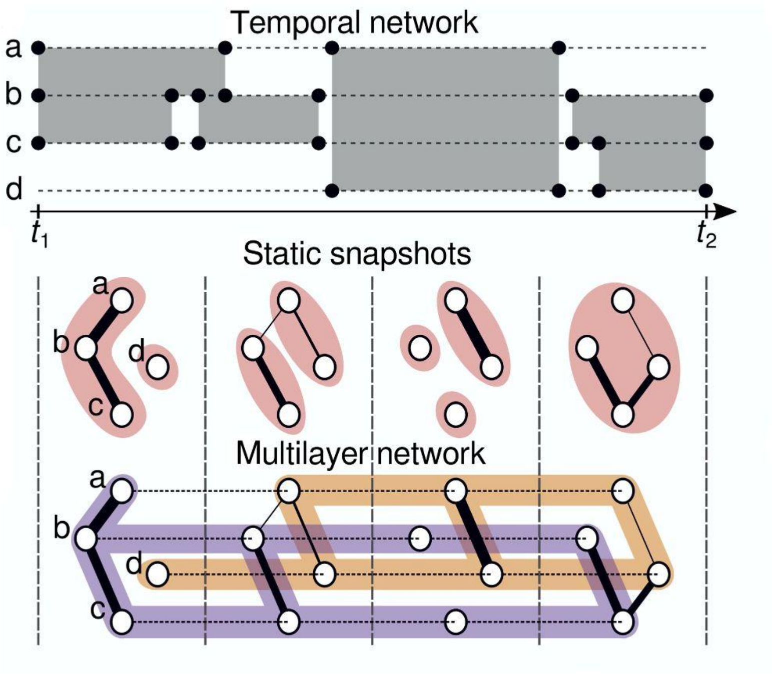


P. Holme and J. Saramäki, Phys. Rep. 519, 97 (2012).

Community detection in temporal networks

Current methods aggregate the temporal dynamics or rely on the assumption of an **underlying stationary process**:

- Multislice generalization of Modularity (Mucha *et al.* 2010)
- Multilayer Infomap (De Domenico *et al.* 2015 & Aslak *et al.* 2018)
- Markov Chain Block Model inference (Piexoto, Rosvall 2017)



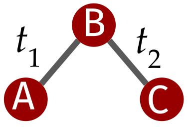
Dynamics-based methods consider a process **decoupled** from the **intrinsic time** of the system under study to guarantee its stationarity.

What is the meaning of a *temporal* community?

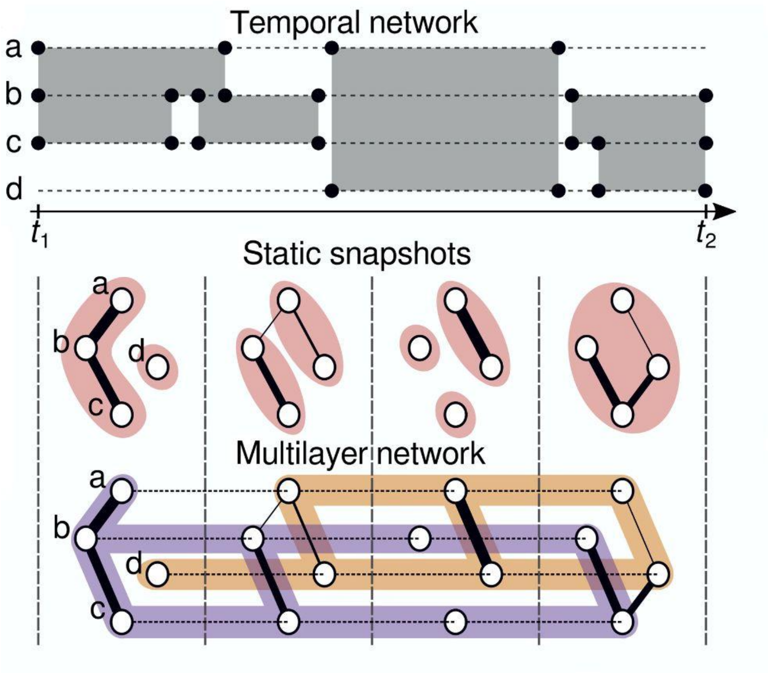
Assortative communities: **densely connected** group of nodes (**symmetric** relation between nodes)

Temporal network:

- Density of connections must be considered over a time range
- Over a time range, relations between nodes are in general **asymmetric**



Asymmetric temporal paths



Idea:

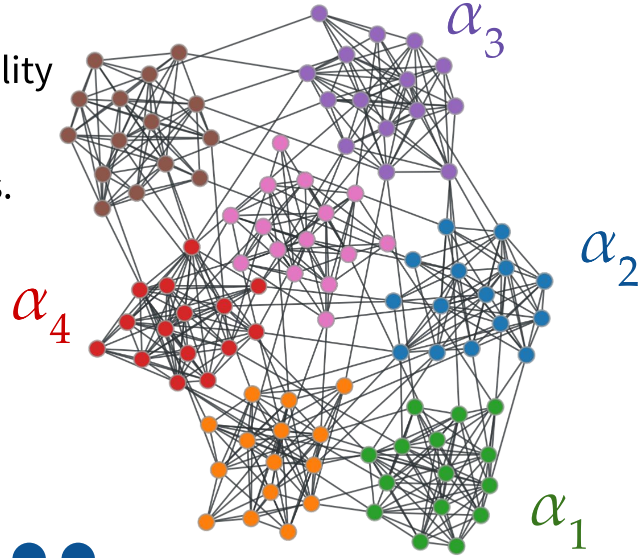
- No aggregation in static snapshots
- Compare the synchronous evolution of a diffusive flow
- Diffusive process **does not necessarily reach a stationary state**

Random walks: a principled approach to study the modular structure of static networks

Can we formulate a quality function that takes into account the probability of random walk (RW) to stay inside communities for long times?

Consider an undirected network and a partition of its nodes in K groups. Associate a real value α_k to each node inside group k, different for each group.

If the partition matches well the community structure of the network, the sequence of α_k values observed by a random walker will have long periods with the same values.



Good partition:

Bad partition:

The sequence of α_k values observed by a random walker can be described by a random process $(Y_t)_{t\in\mathbb{N}}$ for a Discrete-Time RW or $(Y_t)_{t\in\mathbb{R}}$ for a Continuous-Time RW, with $Y_t \in \{\alpha_k \in \mathbb{R} \mid 1 \leq k \leq K\}$

The **autocovariance** of Y_t is a measure of how long Y_t stays the same:

$$\operatorname{cov}[Y_t, Y_{t+\tau}] = \mathrm{E}[Y_t Y_{t+\tau}] - \mathrm{E}[Y_t]\,\mathrm{E}[Y_{t+\tau}]$$

$$= \sum_{k=1}^{K} \left(P(Y_t = \alpha_k \cap Y_{t+\tau} = \alpha_k) - P(Y_t = \alpha_k) P(Y_{t+\tau} = \alpha_k) \right) \alpha_k$$

prob. for a RW to be in the same community at time *t* and *t+τ*

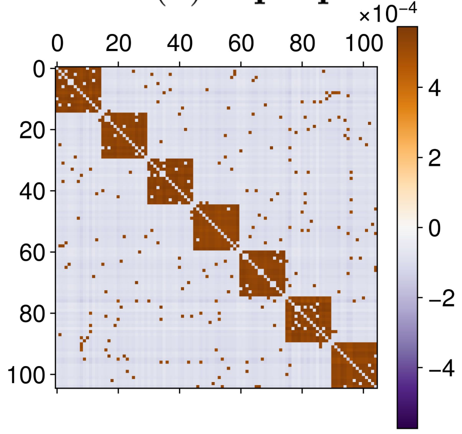
same probability for two independent RW

at stationarity

$$= \sum_{ij} \left(\mathbf{P}^* \mathbf{T}(\tau) - \mathbf{p}^{*\mathsf{T}} \mathbf{p}^* \right)_{ij} \sum_k h_{ik} h_{jk} \alpha_k$$

Covariance matrix

$$\mathbf{P}^* \mathbf{T}(\tau) - \mathbf{p}^{*\mathsf{T}} \mathbf{p}^*$$



Here, $\mathbf{T}(\tau)$ is the RW transition matrix, $\mathbf{p}^*$ is the **stationary** distribution of the RW, $\mathbf{P}^* = \mathrm{diag}(\mathbf{p}^*)$ and h_{ik} encodes the partition ($h_{ik} = 1$ if node i is in community k, $h_{ik} = 0$ otherwise).

The **Markov Stability function** of a graph's partition encoded in $\mathbf{H} = (h_{ik})$ at time τ

$$R(\tau; \mathbf{H}) = \operatorname{trace}\left[\mathbf{H}^{\mathsf{T}} \left(\mathbf{P}^* \mathbf{T}(\tau) - \mathbf{p}^{*\mathsf{T}} \mathbf{p}^* \right) \mathbf{H} \right]$$

measures the quality of the graph's partition in terms of how well it retains random walkers.

Delvenne, Yaliraki, Barahona, PNAS (2010), Lambiotte, Delvenne & Barahona, *IEEE Trans. Net. Sci. Eng.* (2014)

On a connected, undirected network with adjacency matrix $\mathbf{A}$, M edges, and degree vector $\mathbf{k}$:

Markov stability for a DTRW at $t = 1$ is **Modularity**: $R^{\mathrm{DT}}(1; \mathbf{H}) = \frac{1}{2M} \mathrm{trace} \left[\mathbf{H} \left(\mathbf{A} - \frac{\mathbf{k}\mathbf{k}^{\mathsf{T}}}{2M} \right) \mathbf{H}^{\mathsf{T}} \right] = Q$

For a **Continuous-Time RW** with a rate of jumping λ, we have:

Transition matrix: $\mathbf{T}(\tau) = e^{-\lambda \tau \mathbf{L}}$

RW Laplacian: $\mathbf{L} = \mathbf{I} - \mathrm{diag}(\mathbf{k})^{-1} \mathbf{A}$

Stationary state: $\mathbf{p}^* = \mathbf{k}/2M$

Continuous-Time Markov Stability:

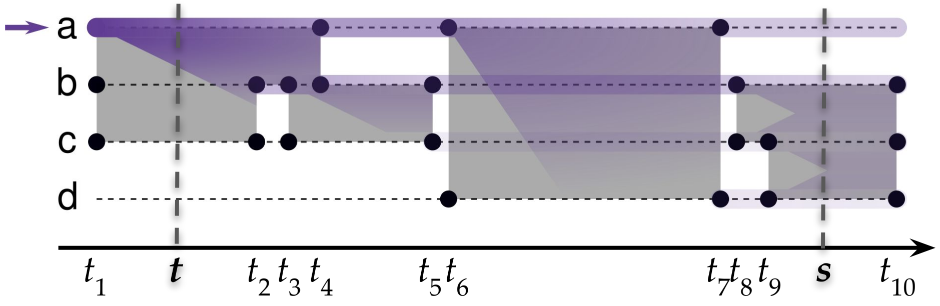
$$R^{\mathrm{CT}}(\tau; \mathbf{H}) = \mathrm{trace} \left[\mathbf{H}^{\mathsf{T}} \left(\mathbf{P}^* e^{-\lambda \tau \mathbf{L}} - \mathbf{p}^{*\mathsf{T}} \mathbf{p}^* \right) \mathbf{H} \right]$$

The time τ plays the role of a **resolution parameter**

Lambiotte, Delvenne & Barahona, *IEEE Trans. Net. Sci. Eng.* (2014)

How to generalize Markov Stability to temporal networks?

Continuous-Time RW with a **rate of jumping** λ on an **evolving network**



inter-event time: $\tau_k = t_{k+1} - t_k$

RW Laplacian at t_k : $\mathbf{L}(t_k)$

Transition probability matrix between consecutive times t_k and t_{k+1} :

$$\hat{\mathbf{T}}(t_k, t_{k+1}) = e^{-\lambda \tau_k \mathbf{L}(t_k)}$$

Transition probability matrix between times t and s:

$$\mathbf{T}(t,s) = \hat{\mathbf{T}}(t, t_2) \left[\prod_{k=2}^{8} \hat{\mathbf{T}}(t_k, t_{k+1}) \right] \hat{\mathbf{T}}(t_9, s)$$

For an initial condition $\mathbf{p}(t)$, we find $\mathbf{p}(s)$ as

$$\mathbf{p}(s) = \mathbf{p}(t)\mathbf{T}(t,s)$$

Problem: as the network is evolving, in general, the RW **does not reach a stationary state.**

Temporal clustering using the RW covariance

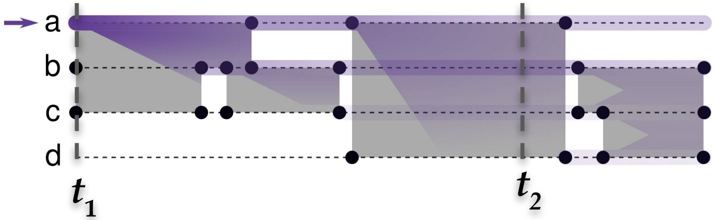
No assumption on the stationarity of the process ⇒ the covariance depends on t_1 and t_2:

$$\operatorname{cov}\left[Y_{t_1}, Y_{t_2}\right] = \mathrm{E}\left[Y_{t_1} Y_{t_2}\right] - \mathrm{E}\left[Y_{t_1}\right] \mathrm{E}\left[Y_{t_2}\right]$$

Covariance matrix:

$$\mathbf{P}(t_1)\mathbf{T}(t_1, t_2) - \mathbf{p}(t_1)^{\mathsf{T}}\mathbf{p}(t_2)$$

element (i,j): joint probability of being on i at t_1 and j at t_2 minus the same probability for two independent random walkers.



Grouping nodes together using this covariance matrix would **compare their state at different times**

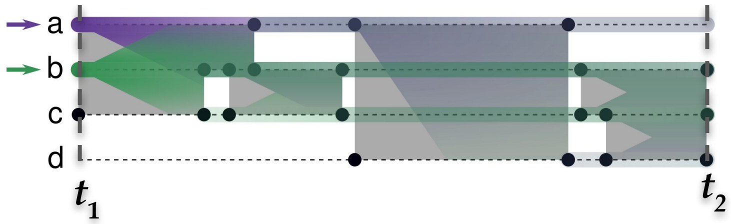
This asynchronous comparison can lead to **asymmetric** relations

How can we compare the **synchronous evolution** of a RW process on temporal network?

Capturing the synchronous evolution of RW

Covariance matrix:

$$\mathbf{P}(t_1)\mathbf{T}(t_1,t_2) - \mathbf{p}(t_1)^\mathsf{T}\mathbf{p}(t_2)$$



We consider the transition matrix of the **inverse process** (for $t_1 < t$):

$$\mathbf{T}^{\mathrm{inv}}(t,t_1) = \mathbf{P}^{-1}(t)\mathbf{T}(t_1,t)^\mathsf{T}\mathbf{P}(t_1)$$ (Bayes' theorem)

$$\mathbf{p}(t)\mathbf{T}^{\mathrm{inv}}(t,t_1) = \mathbf{p}(t_1)$$

Forward covariance ($t_1 < t$)

$$\mathbf{S}_{\mathrm{forw}}(t_1,t) = \mathbf{P}(t_1)\mathbf{T}(t_1,t)\mathbf{T}^{\mathrm{inv}}(t,t_1) - \mathbf{p}(t_1)^\mathsf{T}\mathbf{p}(t_1)$$

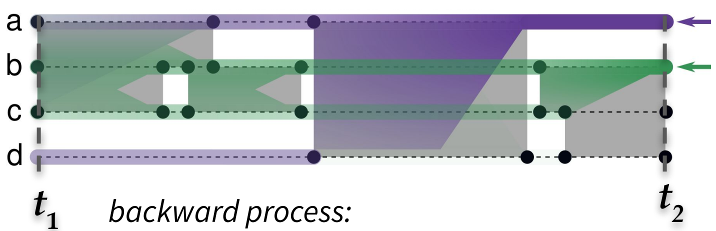
element (i,j): probability that two walkers start on i and j at t_1 and are on the same node at t minus the same probability for two independent random walkers.

synchronous comparison, **symmetric** matrices, **non-stationary** process

Capturing the synchronous evolution of RW

Forward covariance captures how nodes at t_1 are **sources of a similar flow**.

To capture the evolution between two times, we also consider a **backward process**.



backward process:
diffusion on the reversed evolution of the network

Backward covariance ($t < t_2$)

$$\mathbf{S}_{\mathrm{back}}(t_2, t) = \mathbf{P}(t_2)\mathbf{T}_{\mathrm{rev}}(t_2, t)\mathbf{T}_{\mathrm{rev}}^{\mathrm{inv}}(t, t_2) - \mathbf{p}(t_2)^{\mathsf{T}}\mathbf{p}(t_2)$$

Reverse process transition: $\mathbf{T}_{\mathrm{rev}}(t_2, t)$

Backward covariance captures how nodes at t_2 are **sinks of a similar flow**.

Forward and **backward** flow stability functions

For a time range $[t_1, t_2]$, we have two **quality functions**

$$I_{\text{forw}}^{\text{flow}}(t_1, t_2; \mathbf{H}_f) = \text{trace}\left[\mathbf{H}_f^{\mathsf{T}} \int_{t_1}^{t_2} \mathbf{S}_{\text{forw}}(t_1, t)\, dt \mathbf{H}_f\right]$$

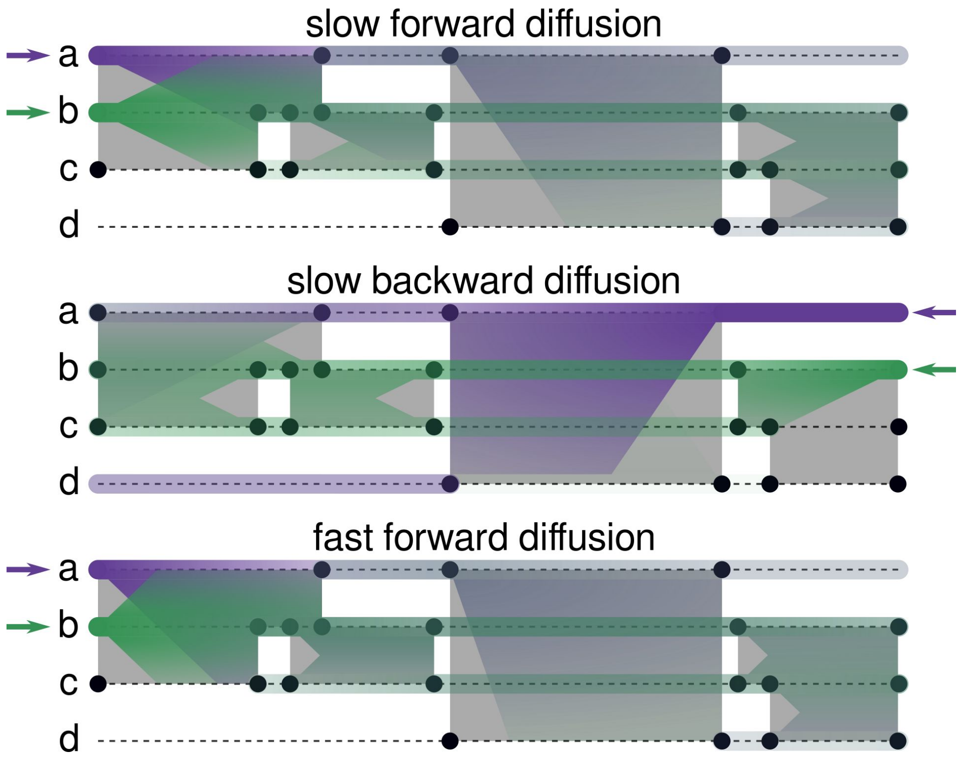
$$I_{\text{back}}^{\text{flow}}(t_1, t_2; \mathbf{H}_b) = \text{trace}\left[\mathbf{H}_b^{\mathsf{T}} \int_{t_2}^{t_1} \mathbf{S}_{\text{back}}(t_2, t)\, dt \mathbf{H}_b\right]$$

The best **forward** ($\mathbf{H}_f$) and **backward** ($\mathbf{H}_b$) partitions:

- Best clustering of “sources” and “sinks” of the a diffusive process coupled with the network evolution
- Does not require the process to reach a stationary state
- Capture the time ordering of events
- Symmetric relations between nodes of a same community
- Asymmetry due to the network evolution is captured by using two partitions

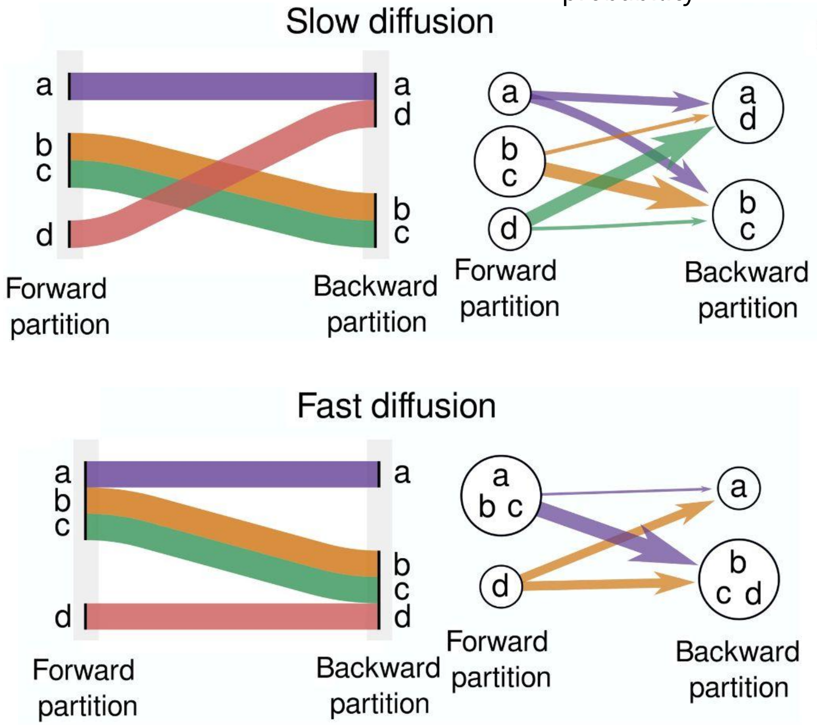
Bovet, Delvenne & Lambiotte. *Sci. Adv.* (2022)

Flow stability partitions gives a point of view



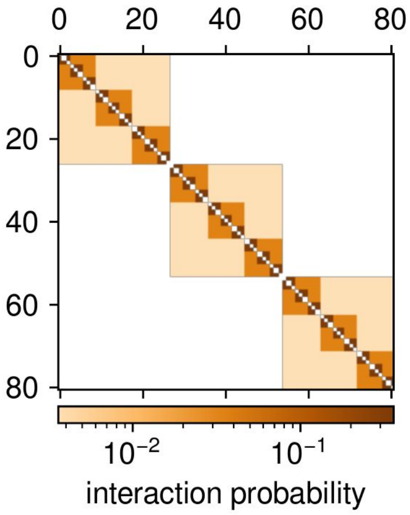
Linking by node overlap

Linking by transition probability

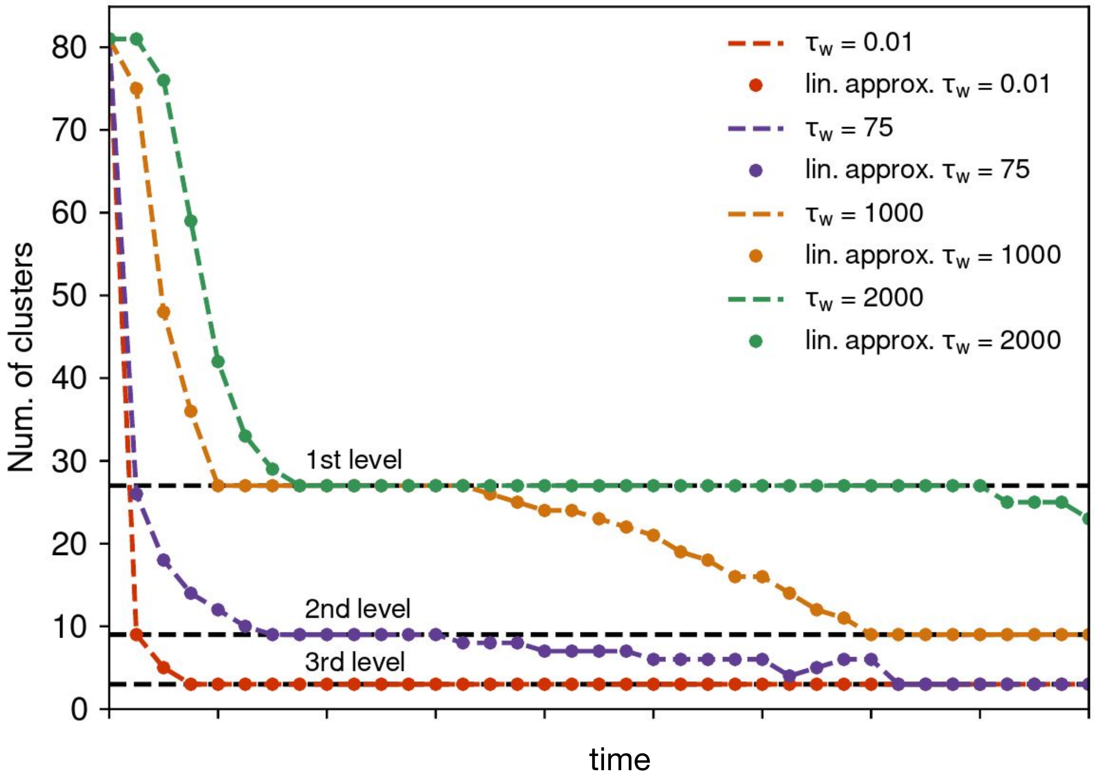


Dynamic scale for hierarchical clustering

Random walk rate (waiting time) plays the role of a **resolution parameter**

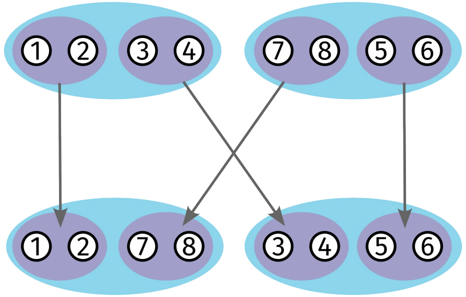


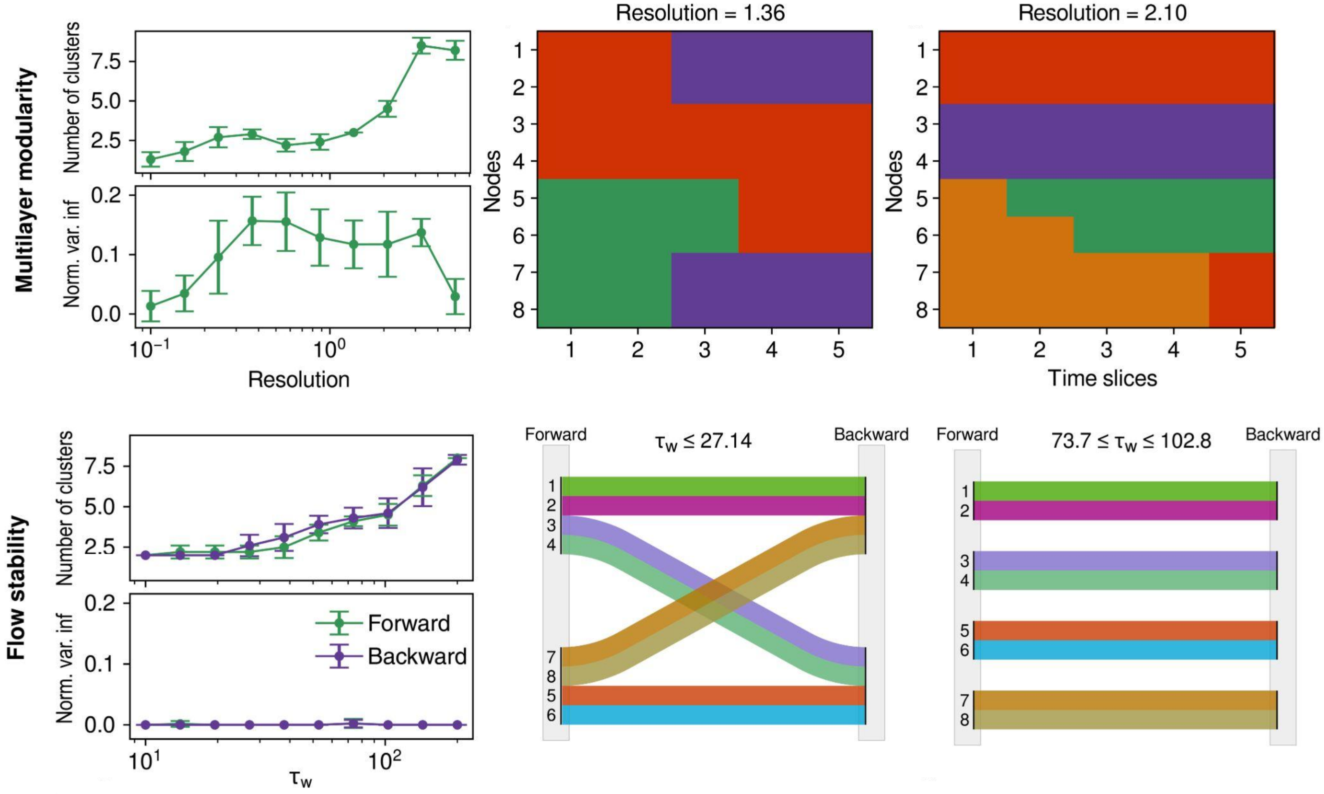
3 levels stationary temporal block model



Dynamic scale for capturing changes

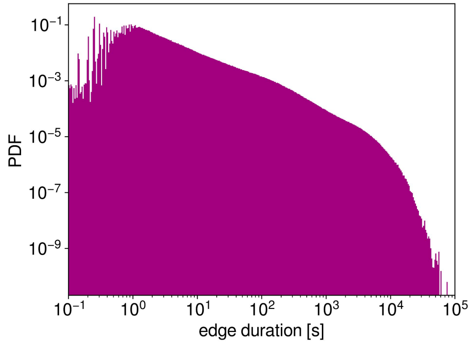
Continuously changing temporal network



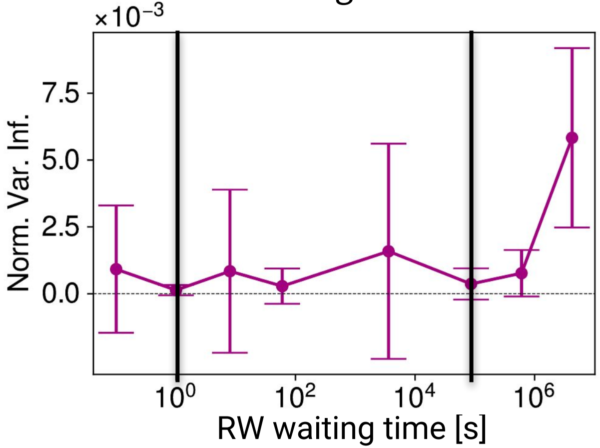


Temporal contact network of free-living wild mice

- 437 wild mice
- 2 months (Feb 28th to May 1st 2017)
- Millisecond resolution
- 5.7M edges
- Weekly intervals
- Uniform initial conditions



Variation of Information for 50 runs of Louvain algorithm

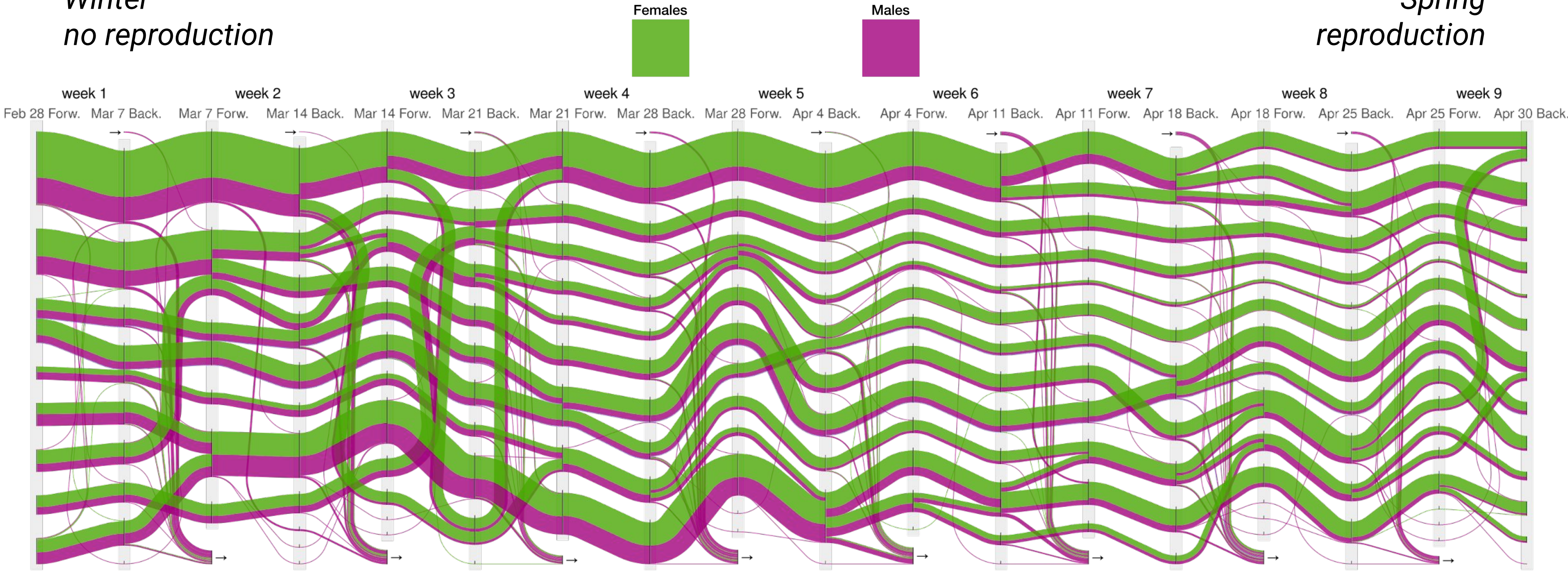


B. König *et al.*, *Anim. Biotelemetry* 3, 39 (2015).

Temporal contact network of free-living wild mice: week per week dynamics

Winter
no reproduction

Spring
reproduction

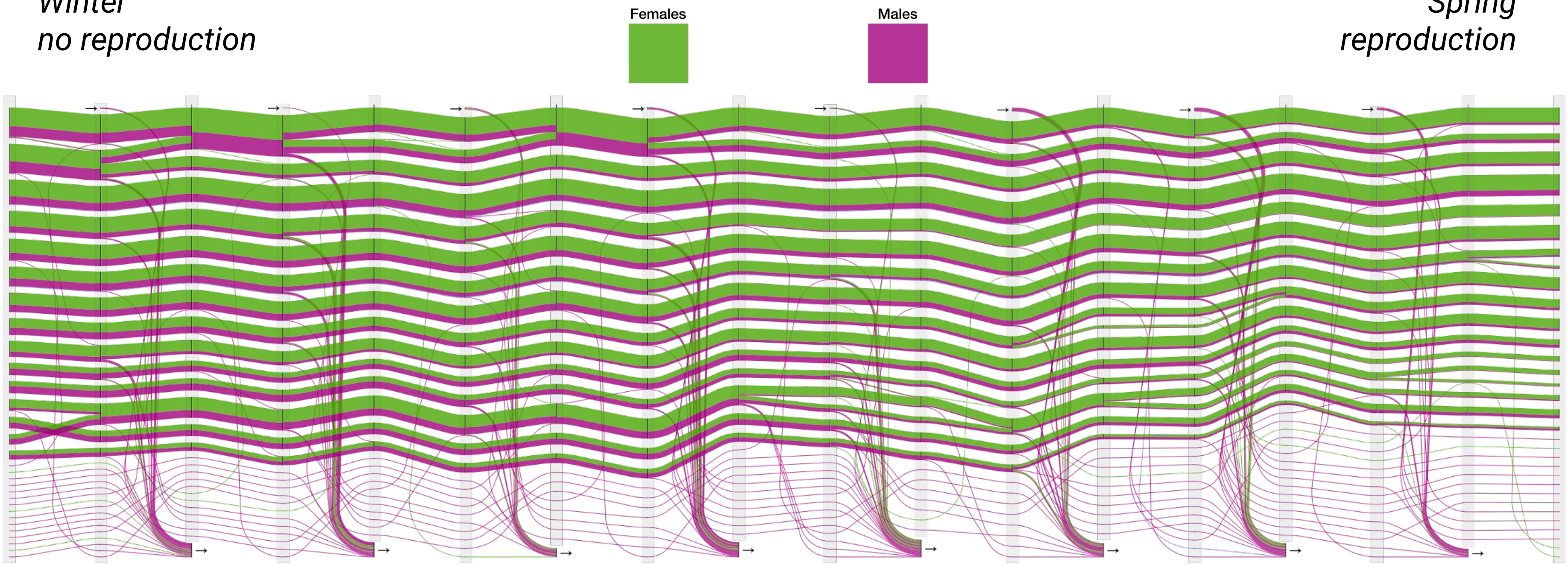


RW waiting time = 1s

Existence of two simultaneous dynamics
finer scale: stable communities

Winter
no reproduction

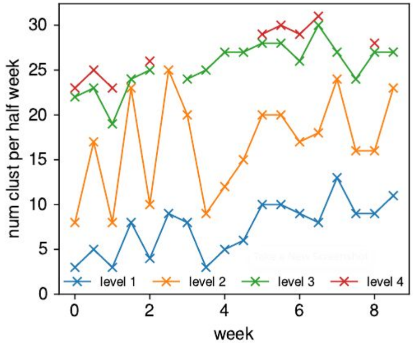
Spring
reproduction

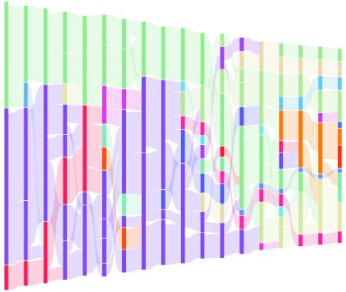


RW waiting time = 24h

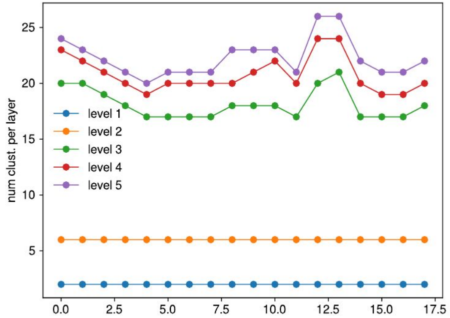
The RW rate is a dynamic scale consistent across slices

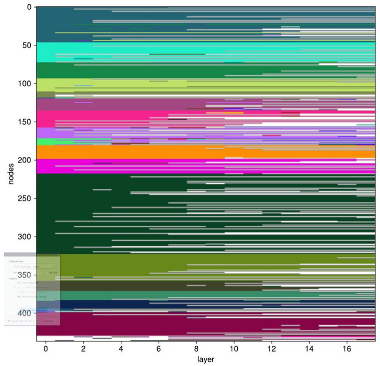
Hierarchical Infomap on each slice + community tracking



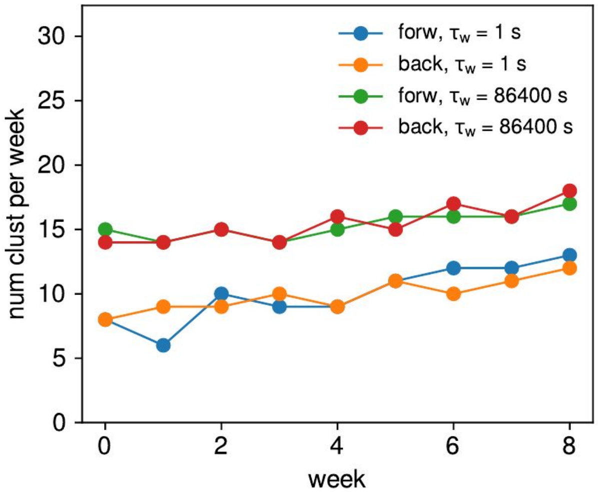


Hierarchical multilayer Infomap





Flow stability

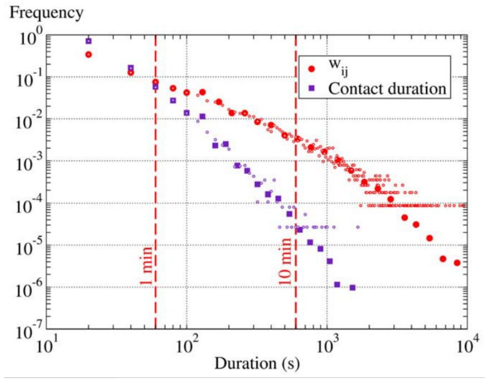


We look at the same dynamic scale in each slices: smooth variation

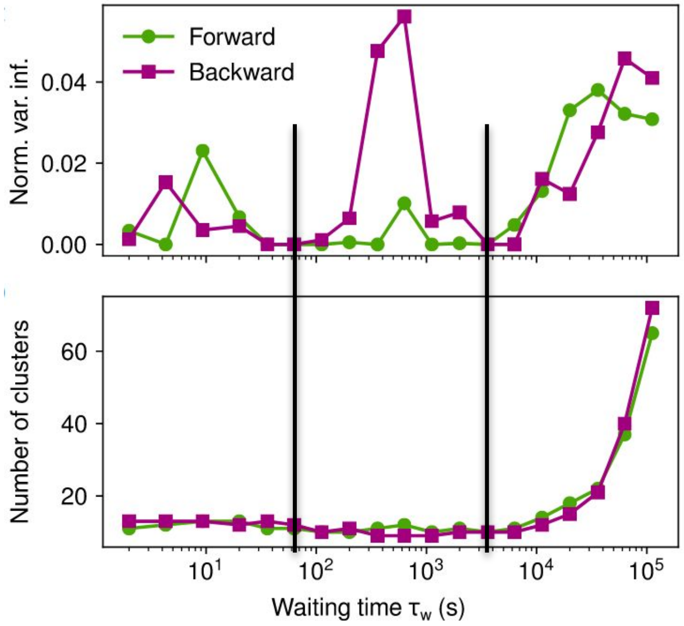
Rosvall *et al.*, PLOS One (2011), De Domenico *et al*. Phys. Rev. X (2015)

Primary school contacts network

- 232 children + 10 teachers
- 5 grades, each separated in two classes
- 2 days
- 20 second resolution
- Uniform initial conditions

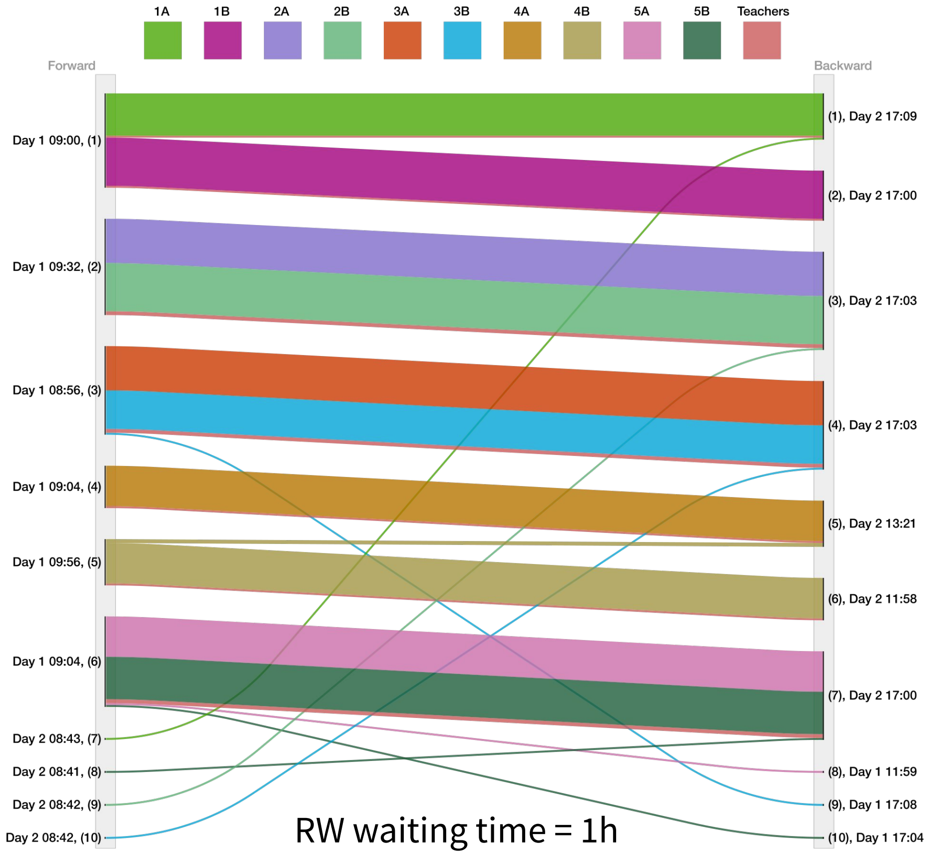


Variation of Information for 50 runs of Louvain algorithm

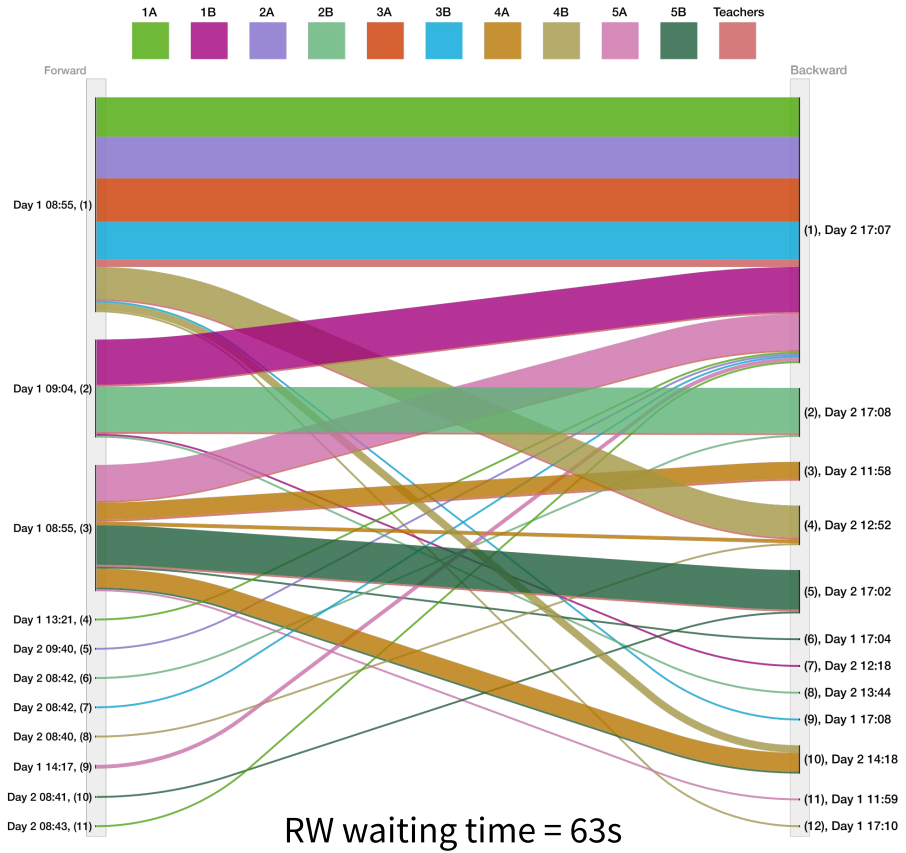


Stehlé, J. et al. PLoS One 6, e23176 (2011)

Primary school: detect temporal structures at different dynamic scales



RW waiting time = 1h

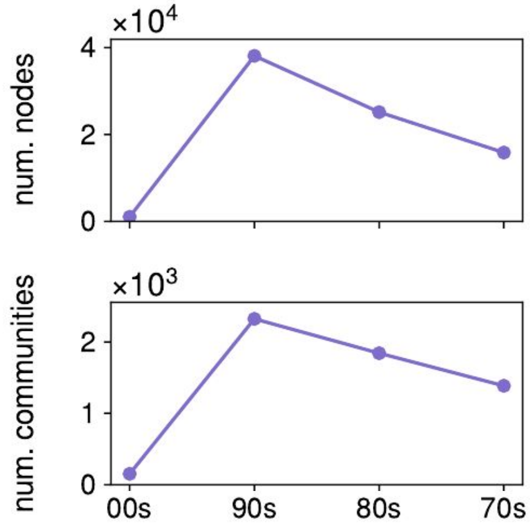


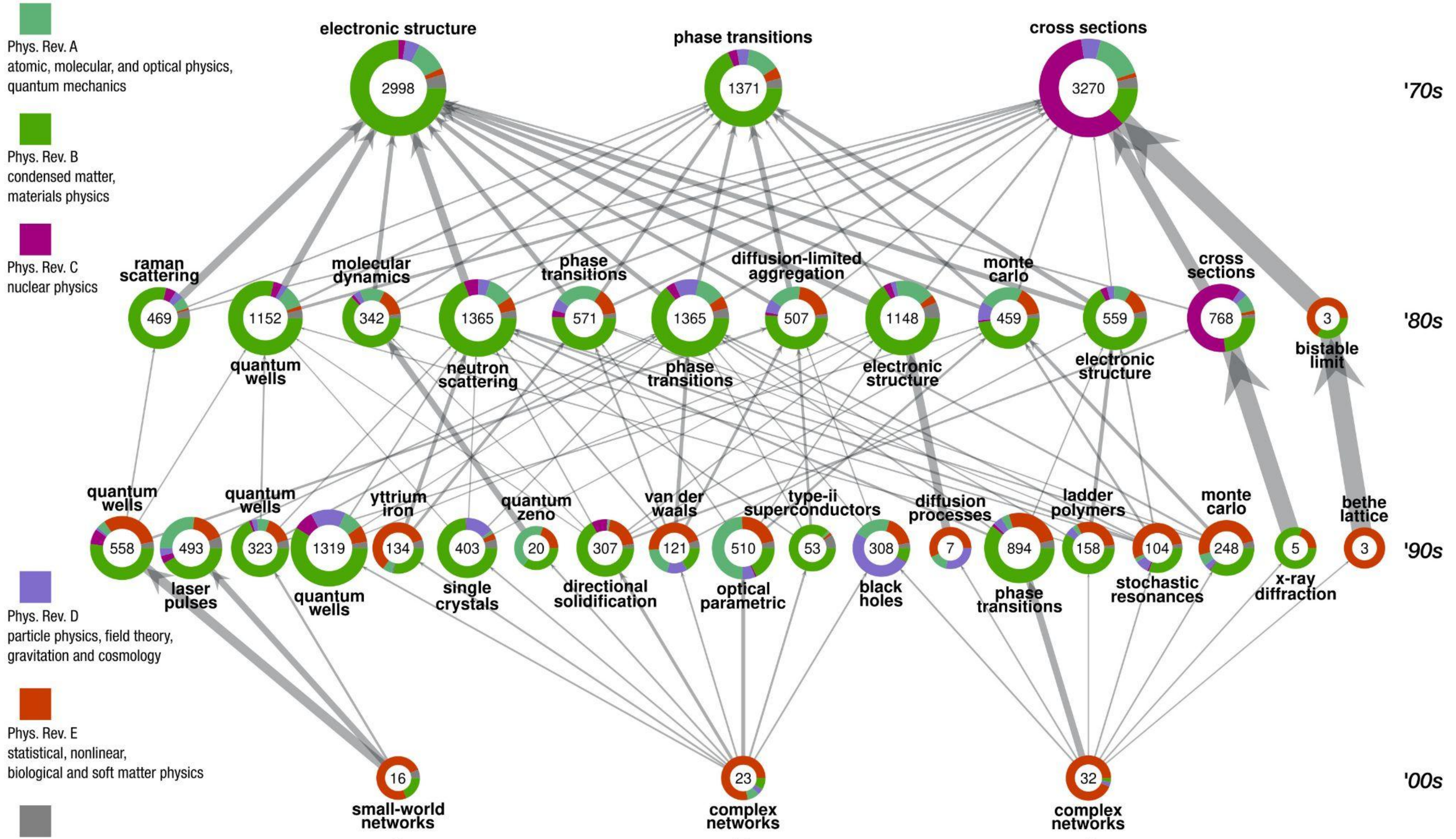
RW waiting time = 63s

Using **non-uniform initial distribution**: uncovering the physical influences of network scientists

Unique ability of the method to cluster the **non-stationary flow** representing the diffusion of ideas in the APS co-authorship network.

- Random walk starting on the authors from the network science community of the 2000s
- Propagation backward in time until 1970
- **Backward flow stability** clustering applied to each decade
- Backward communities are **linked** by computing the transition probability between communities of different decade
- We find the “ancestor” communities of the 2000s authors





A
USA/Hungary

B
UK/Finland/Spain

C
Italy/USA/Spain

Non-stationary processes on static directed networks

An issue for RW-based community detection method is the absence of a unique non-trivial stationary state on static directed networks.

Usually, a PageRank teleportation is introduced in order to make the RW ergodic.

“Flow” communities are found, but the “direction” of the flow is lost.

Flow stability can capture the **asymmetric** relations between communities in terms of flow:

Forward process:

- Iteratively remove nodes with zero out-degree: $\mathbf{A}_f$
- Cluster forward covariance

Backward process:

- Iteratively remove nodes with zero in-degree: $\mathbf{A}_b$
- Diffusive process on the reversed network
- Cluster backward covariance

Forward Laplacian

$$\mathbf{L}_\mathrm{f} = \mathbf{I} - \mathbf{D}_\mathrm{out}^{-1}\mathbf{A}_\mathrm{f}$$

Forward Transition

$$\mathbf{T}_\mathrm{f}(t) = e^{-t\mathbf{L}_\mathrm{f}}$$

$$\mathbf{S}_\mathrm{forw}(t) = \frac{1}{N_\mathrm{f}}\mathbf{T}_\mathrm{f}(t)\mathbf{T}_\mathrm{f}^\mathrm{inv}(t) - \frac{1}{N_\mathrm{f}^2}\overleftrightarrow{\mathbf{1}}$$

Backward Laplacian

$$\mathbf{L}_\mathrm{b} = \mathbf{I} - \mathbf{D}_\mathrm{in}^{-1}\mathbf{A}_\mathrm{b}^\mathsf{T}$$

Backward Transition

$$\mathbf{T}_\mathrm{b}(t) = e^{-t\mathbf{L}_\mathrm{b}}$$

$$\mathbf{S}_\mathrm{back}(t) = \frac{1}{N_\mathrm{b}}\mathbf{T}_\mathrm{b}(t)\mathbf{T}_\mathrm{b}^\mathrm{inv}(t) - \frac{1}{N_\mathrm{b}^2}\overleftrightarrow{\mathbf{1}}$$

The two partitions provide a **co-clustering** of the network

Rohe, Qin & Yu, *PNAS* (2016)

Flow stability describes the flow of users in Telegram

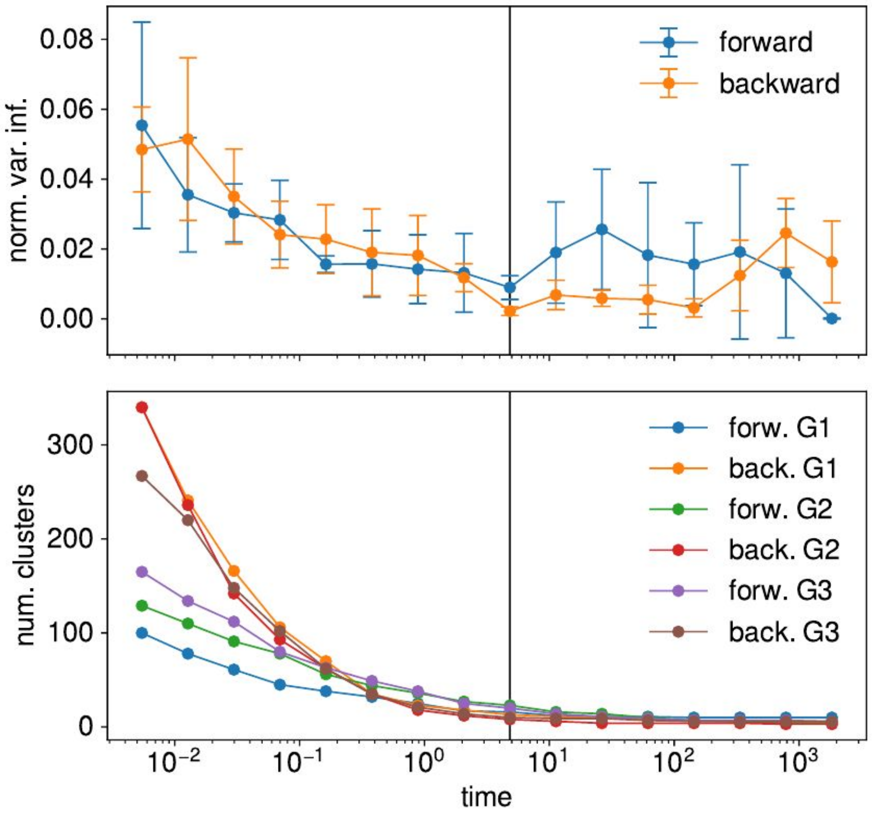
7 million messages from **12,564 channels/groups** related to the UK far-right

⇒ 3 weighted directed network where edges represent potential flow of users: mentions, links or forwards

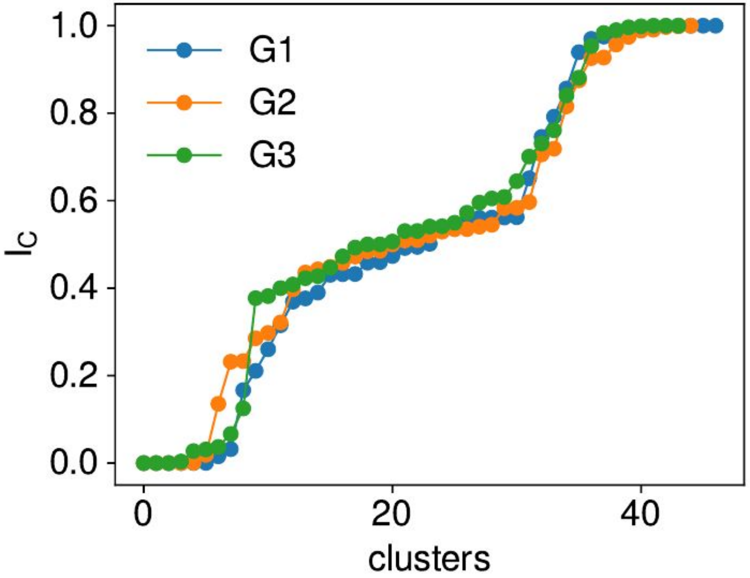
Best **forward** and **backward** partitions provide a clustering in terms of "sources" and "sinks"

Final partition: intersection where they overlap + union where they don't

Diffusion time as a **resolution parameter**



Different classes of flow stability communities



For each community c, compute its 'inness':

$$I_c = \sum_{i \in c} \frac{s_i^{in}}{s_i^{out} + s_i^{in}}$$

Classify communities as

- **Upstream** if $I_c < 0.2$
- **Downstream** if $I_c > 0.8$
- **Core** otherwise

Flow structure of the Telegram network

Core communities

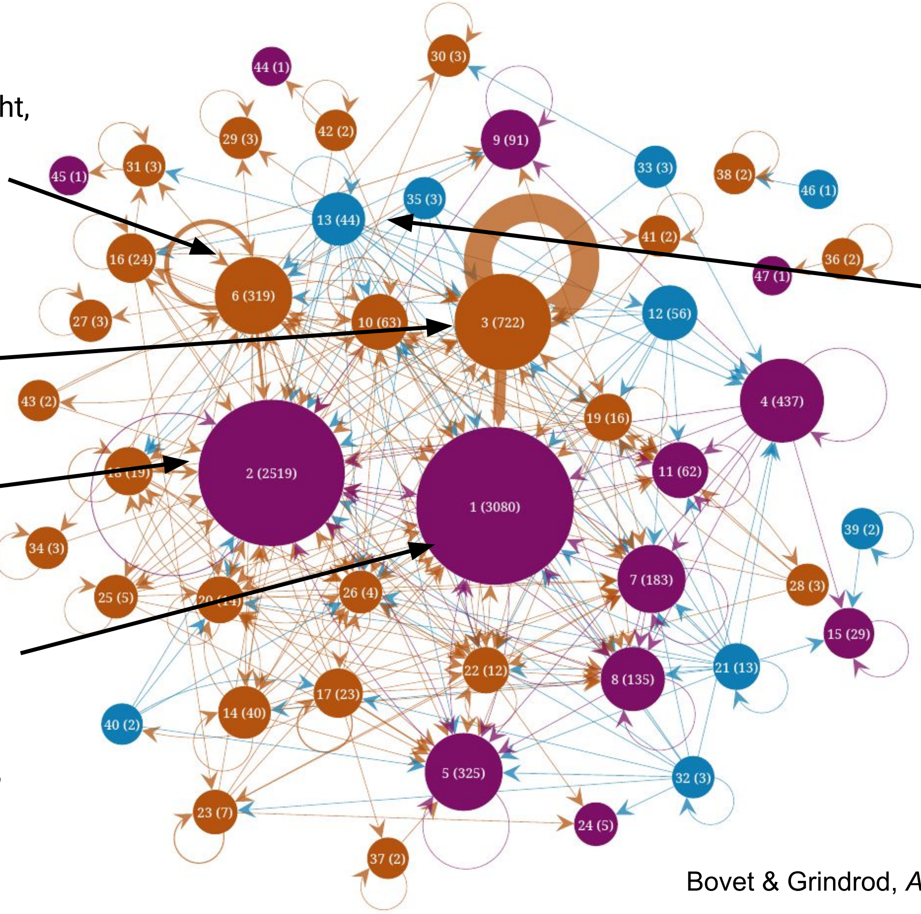
Far-right channels:
randomanonch, Thecelticempire, WhiteIsRight, sgmeme, BloodAndHonour, NazBol, MiloOfficial, shitpost, toalibertarian, TommyRobinsonNews, pol_4chan, HansTerrorwave, AntifaPublicWatch

Russian news/commentary:
go338, karaulny, kbrvdvkr, rt russian, stormdaily, bbbreaking

Downstream communities

UK/US right wing news/politics:
realdonaldtrump, breaking911, ReutersWorldChannel, dailyredpill, AltMemes, khamenei_ir,

Russian news/politics:
kremlin_mother_expert, sexy_moscow, solarstorm, TJournal, nourlnews, varlamovuranews, crimeainform,



Sep. 2015 to June 2019

Upstream community **group chats:**
LeHumbleKekVerse*, brexiteerschatlounge*, judenpresse_archive, q_anons*, CrypticCoinVIP*, fitinorfuckoff*

Bovet & Grindrod, *Appl Netw Sci* 7, 76 (2022).

Outlook

The dynamics of complex systems arises from the interaction between several temporal processes.

Temporal networks and community detection allow us to extract a simplified view of their dynamics.

The **flow stability method** defines communities in terms of a RW evolving with the network.

By varying the rate of the RW we can dicover different **dynamic scales**.

Opens the door to define **new concepts for temporal networks** in terms of RW and flows.

A. Bovet, J.-C. Delvenne, R. Lambiotte
Flow stability for dynamic community detection
Science Advances **8** eabj3063 (2022)
Code: https://github.com/alexbovet/flow_stability

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